Trigonometric functions of any angle pdf

Trigonometric Functions: An angle having measure greater than but less than is called an acute angle. Consider a right angled triangle ABC with right angle at B. The side which is opposite to right angle is known as hypotenuse, the side opposite to angle A is called perpendicular for angle A and the side opposite to third angle is called base for angle A. Any ratio of two of the sides depends only on the measure of angle A, for if we take a lager or smaller right angle A' B' C' with A = A' then , (as these triangles are similar) Thus any ratio of the lengths of two sides of the triangle is completely determined by angle A alone and is independent of the size of the triangle. There are six possible ratios that can be formed from the three sides of a right angled triangle. Each of them has been given a name as follows. Definitions: The abbreviations stand for sine, cosine, tangent, cotangent, secant and cosecant of A respectively. These functions of angle A are called trigonometrical functions or trigonometrical ratios. Trigonometrical functions of any angle: Let A be a given angle with specified initial ray. We introduce rectangular coordinate system in the plane with the vertex of angle A as the origin and the initial ray of angle A as the positive ray of the x-axis. We choose any point P on the terminal ray of angle A. Let the coordinates of P be (x, y) and its distance from the origin be r, then we define These quantities are functions of the angle A alone. They do not dependon the choice of the point P and the terminal ray for if we choose a different point P' (x', y') on the terminal ray of A at a distance r' from the origin, it is clear that x' and y' will have the same sign as those of x and y respectively and because of similar and , , , , etc. will be equal to , , , respectively (r being always positive). Also any trigonometrical function of an angle is equal to the same trigonometrical function of any angle , where n is any integer since all these angles will have the same terminal ray. For example, . After the coordinates system has been introduced, the plane is divided into four quadrants. An angle is said to be in that quadrant in which its terminal ray lies. For positive acute angles this definition gives the same result as in case of a right angled triangles since x and y are both positive for any point in the first quadrant and consequently are the length of base and perpendicular of the angle A. In 1st quadrant , , , , are all positive as x, y are positive. In 2nd quadrant x is negative and y is positive therefore, only and are positive. In 3rd quadrant both x and y are negative, therefore only and are positive. In 4th quadrant x is positive and y is negative, therefore only   and   are positive. Limits of the values of trigonometrical functions: and therefore, Relation between the trigonometrical ratios of angles: Let O be the vertex of an angle A, OX its initial ray and OP its terminal ray. Let P (x, y) be a point on the terminal ray. Let PL be perpendicular to OX'. Let OP = r then Now, I. Proof: II. Proof: III. Proof: IV. Proof: similarly, and Important Results ; ; ; ; ; Note: Positive or negative sign before the root depends on the quadrant in which A lies. Examples: Ex. – 1. Prove that Sol. – L.H.S.   R.H.S. Ex. – 2. If . Then prove that the value of each side is Sol. – Let   …(i) then, …(ii) Multiplying (i) and (ii) we get or,   Hence each side Ex. – 3. If , prove that Sol. – Given, Now, L.H.S. R.H.S. Exercise: Prove the following trigonometrical identities. 1. . 2. . 3. . 4. . 5. . 6. . 7. If , prove that . 8. If , prove that . 9. If then prove that the value of each . 10. If , find the value of , , . Porvided . 11. If , , find , and . 12. If , find . P.S. – Please find the answers here. Precalculus by Richard Wright Previous Lesson Table of Contents Next Lesson Are you not my student andhas this helped you? Summary: In this section, you will: Evaluate trigonometric functions of any angle. Find reference angles. Figure 1: London Eye. credit (pixabay/skeeze) The London Eye is a farris wheel with a diameter of 394 feet. By combining the ideas of the unit circle and right triangles, we can describe the location of any capsule on the Eye with trigonometry. Trigonometric Functions of Angles on Circles with r ≠ 1 In lesson 4.2, we looked at the unit circle. In lesson 4.3 we explored right triangle trigonometry. To combine these ideas, consider a circle where r ≠ 1. Pick a point on the circle. A right triangle can be drawn to the point where one acute angle is at the point, the other acute angle is at the origin, and the right angle is on the x-axis. Figure 2: Right triangle drawn to a point on a circle. By comparing the unit circle formulas and the right triangle formulas, we can develop the formulas for any angle. For example, consider sin θ. sin θ = y Unit Circle sin θ = opp hyp Right Triangle sin θ = y r Apply the right triangle formula for the acute angle by the origin. Notice the last equation matches the unit circle formula with r = 1. All the unit circle formulas can be similarly modified. sin θ = y r csc θ = r y cos θ = x r sec θ = r x tan θ = y x cot θ = x y where θ is an angle in standard position with point (x, y) on the terminal side and r=x2+y2 Let (−4, 3) be a point on the terminal side of angle θ. Evaluate the six trigonometric functions of θ. Figure 3 Solution Find r. r = x 2 + y 2 r = - 4 2 + 3 2 r = 5 Now use the trigonometric formulas. sin θ = y r = 3 5 csc θ = r y = 5 3 cos θ = x r = - 4 5 sec θ = r x = - 5 4 tan θ = y x = - 3 4 cot θ = y r = - 4 3 If (4, −8) is a point on the terminal side of angle α in standard position, evaluate the six trigonometric functions of α. Answers sin α = - 2 5 5 csc α = - 5 2 cos α = 5 5 sec α = 5 tan α = - 2 cot α = - 1 2 Evaluate cos 270° and csc π. Solution 270° and π radians terminal sides are both on an axis. Start by choosing a point on the terminal sides of the angle. Figure 4: Points for quadrantal angles. Now apply the trigonometric formulas with r = 1. cos θ = x r csc θ = r y cos 270 ° = 0 1 = 0 csc π = 1 0 = undefined Evaluate sin 90° and cot 0. Answer 1; undefined Signs of Trigonometric Functions in the Quadrants By filling in the negative signs for x and y from the quadrants into the trigonometric formulas and pattern develops. For example consider sine and cosine in quadrant II where the x is negative and y is positive. sin θ = y r Since the y and r are both positive, sine is positive. cos θ = x r Since the x is negatvie and r are both positive, cosine is negative. All the trigonometric functions' signs can be similarly determined for all four quadrants. Figure 5 shows which trigonometric are positive in each quadrant. Figure 5: Positive trigonometric functions in each quadrant. If cos θ = - 8 17 and sin θ < 0, find tan θ and csc θ. Solution cos θ = x r = - 8 17 Since the r is always positive, r = 17 and x = −8. Use the Pythagorean Theorem to find y. x 2 + y 2 = r 2 - 8 2 + y 2 = 17 2 y 2 = 225 y = ± 15 Since we are told that sin θ < 0 and sinθ=yr, y must be negative. So, y = −15. Now we know x = −8, y = −15, and r = 17. Since both x and y are negative, the angle terminates in quadrant III where tan θ and cot θ are positive. We could have also looked for a quadrant where both sin θ and cos θ were negative which is quadrant III. Now fill in the trigonometric formulas. sin θ = y r = - 15 17 csc θ = r y = - 17 15 cos θ = x r = - 8 17 sec θ = r x = - 17 8 tan θ = y x = 15 8 cot θ = y r = 8 15 If sinθ=-53 and cos θ > 0, find tan &theta and cos θ. Answers tan θ = - 5 2 ; cos θ = 2 3 Reference Angles Since the formula from the unit circle and the right triangles give the same expressions for Example 1, the acute angle by the origin in the triangle and the angle in standard position must give the same values of the trigonometric functions. Those acute angles are useful and called reference angles. The reference angle is the angle between the terminal side of an angle in standard position and the nearest x-axis. Reference angles are always < π2. The values of the trigonometric functions of the angle in standard position = values of the trigonometric functions of the reference angle with the appropriate negative signs for the quadrant. Figure 6: α is the Reference Angle To find the reference angle Determine the measure of the angle whose terminal side is on the x-axis nearest the terminal side of the given angle. Subtract the measures of the given angle and the x-axis angle. Find the reference angle for a) 7π6, b) 2π3, c) π4, and d) 7π4. Solution a. It is easiest to start by stetching a graph of the angle in standard position like in Figure 7. The x-axis angle nearest the terminal side of angle θ is π. Subtract π and 7π6 to find the reference angle. 7 π 6 - π 7 π 6 - 6 π 6 = π 6 The reference angle is π6. Figure 7: 7π6 b. The graph of 2π3 is in Figure 8. The x-axis angle nearest the terminal side of angle θ is π. Subtract π and 2π3 to find the reference angle. π - 2 π 3 3 π 3 - 2 π 3 = π 3 The reference angle is π3. Figure 8: 2π3 c. The graph of π4 is in Figure 9. The x-axis angle nearest the terminal side of angle θ is 0. Subtract 0 and π4 to find the reference angle. π 4 - 0 = π 4 The reference angle is π4. Figure 9: π4 d. The graph of 7π4 is in Figure 10. The x-axis angle nearest the terminal side of angle θ is 2π. Subtract 2π and 7π4 to find the reference angle. 2 π - 7 π 4 8 π 4 - 7 π 4 = π 4 The reference angle is π4. Figure 10: 7π4 Find the reference angle for 5π3. Answer π3 Trigonometric Functions of Real Numbers All the ideas from this lesson can be combined to evaluate trigonometric functions of any real number. Evaluate Trigonometric Functions of Any Real Number Consider the number to be an angle, θ. If θ is not between 0 and 2π, find a coterminal angle that is by adding or subtracting 2π. Find the reference angle. Evaluate the trigonometric function of the reference angle using special right triangles (lesson 4-04) or the unit circle (lesson 4-02). Apply negative sign as needed based on the quadrant θ is in. Evaluate a) cos7π6, b) sin2π3, c) tan13π4, and d) sin-7π4. Solution 7π6 is between 0 and 2π, so we can start finding the reference angle. In Example 4 we found the reference angle of 7π6 is π6. Evaluate the function of the reference angle using special right triangles or the unit circle. cosπ6=32 7π6 is in quadrant III and cosine is negative in quadrant III, so cos7π6=-32. 2π3 is between 0 and 2π, so we can start finding the reference angle. In Example 4 we found the reference angle of 2π3 is π3. Evaluate the function of the reference angle using special right triangles or the unit circle. sinπ3=32 2π3 is in quadrant II and sine is positive in quadrant II, so sin2π3=32. 13π4 is not between 0 and 2π, so begin by finding a coterminal angle between 0 and 2π. 13 π 4 - 2 π = 13 π 4 - 8 π 4 = 5 π 4 Find the reference angle. 5π4 is in quadrant III and closest x-axis angle is π. 5 π 4 - π = 5 π 4 - 4 π 4 = π 4 Evaluate the function of the reference angle using special right triangles or the unit circle. tan π 4 = 1 13π4 which is coterminal with 5π4 is in quadrant III and tangent is posiive in quandrant III, so tan13π4=1. -7π4 is not between 0 and 2π, so begin by finding a coterminal angle between 0 and 2π. - 7 π 4 + 2 π = - 7 π 4 + 8 π 4 = π 4 Find the reference angle. π4 is in quadrant I and closest x-axis angle is 0. π 4 - 0 = π 4 Evaluate the function of the reference angle using special right triangles or the unit circle. sin π 4 = 2 2 -7π4 which is coterminal with π4 is in quadrant I and sine is posiive in quandrant I, so sin-7π4=22. Evaluate cos5π3. Answer ½ If sinθ=-23 and θ terminates in quadrant IV, find tan θ. Solution One method to solve this problem is to sketch a right triangle in the specified quadrant with an acute angle at the origin and right angle on the x-axis. The sides of the triangle are from the given trigonometric function. Since sinθ=yr=-23. The r is always positive so r = 3 and y = −2. By using the Pythagorean Theorem, find x. x 2 + y 2 = r 2 x 2 + - 2 2 = 3 2 x = ± 5 Since the triangle is in quadrant IV, the x value is positive and x=5 Figure 11 Now evaluate tan θ using the right triangle. tan θ = y x = - 2 5 = - 2 5 5 If tan α = −2 and α terminates in quadrant II, find sin α. Answer 255 sin θ = y r csc θ = r y cos θ = x r sec θ = r x tan θ = y x cot θ = x y where θ is an angle in standard position with point (x, y) on the terminal side and r=x2+y2 Reference Angles The reference angle is the angle between the terminal side of an angle in standard position and the nearest x-axis. Reference angles are always < π2. The values of the trigonometric functions of the angle in standard position = values of the trigonometric functions of the reference angle with the appropriate negative signs for the quadrant. Figure 6: α is the Reference Angle To find the reference angle Determine the measure of the angle whose terminal side is on the x-axis nearest the terminal side of the given angle. Subtract the measures of the given angle and the x-axis angle. Evaluate Trigonometric Functions of Any Real Number Consider the number to be an angle, θ. If θ is not between 0 and 2π, find a coterminal angle that is by adding or subtracting 2π. Find the reference angle. Evaluate the trigonometric function of the reference angle using special right triangles (lesson 4-04) or the unit circle (lesson 4-02). Apply negative sign as needed based on the quadrant θ is in. Evaluate the six trigonometric functions based on the given point on the terminal side of an angle in standard position. (3, −5) (−2, −7) Evaluate the six trigonometric functions of the given angle. π2 2π Evaluate the function of θ. If sinθ=15 and θ is in quadrant II, find a) cos θ and b) tan θ. If secθ=43 and θ is in quadrant IV, find a) sin θ and b) csc θ. If tanθ=-34 and sin θ > 0, find a) cos θ and b) sec θ. If cosθ=-817 and tan θ < 0, find a) sin θ and b) cot θ. Find the reference angle of the given angle. 6π5 4π7 -8π9 15π4 Evaluate the given trigonometric functions using reference angles. sin3π4 tan11π6 cos5π4 Mixed Review (4-04) Let θ be an acute angle. Use the given function value with trigonometric identities to evaluate the given function. If csc θ = 2, find a) cot θ and b) sin θ. (4-04) A student is standing on the third floor of a building 30 feet above the ground. There are two kids on a lawn playing catch with a frisbee. The angles of depression from the student in the building to the kids are 45° and 55°. How far apart are the students? (4-03) Use special right triangles to evaluate a) sinπ3 and b) cotπ4. (4-02) Use the unit circle to evaluate a) cosπ6 and b) tan7π6. (4-01) a) Draw the 17π6 in standard position and find a b) positive and c) negative coterminal angle. sin θ = - 5 34 34 , cos θ = 3 34 34 , tan θ = - 5 3 , csc θ = - 34 5 , sec θ = 34 3 , cot θ = - 3 5 sin θ = - 7 53 53 , cos θ = - 2 53 53 , tan θ = 7 2 , csc θ = - 53 7 , sec θ = - 53 2 , cot θ = 2 7 sin θ = 1 , cos θ = 0 , tan θ = undefined , csc θ = 1 , sec θ = undefined , cot θ = 0 sin θ = 0 , cos θ = 1 , tan θ = 0 , csc θ = undefined , sec θ = 1 , cot θ = undefined -245; -2424 -74; -477 -45; -54 1517; -815 π5 3π7 π9 π4 22 -33 -22 3; ½ about 9 ft 32; 1 32; 33 ; 5π6; -7π6 Previous Lesson Table of Contents Next Lesson

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